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On Pre-Open And M Pre-open Functions

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Abstract: *In this study, the concepts of pre-open, M pre-open and M pre-closed function given in were introduced and their characterizations were investigated. We obtained the characterizations of M pre-open functions. Moreover we gave the concept of M pre-homeomorphism and characterized this concept.*

Key Words: **pre-open set, semi-open set, pre neighbourhood, pre-open function, M pre-open function, M pre-homeomorphism.**

Özet: Bu çalışmada, ön-açık , M ön-açık and M ön-kapalı fonksiyon kavramları sunuldu ve bu kavramların karakterizasyonları incelendi. Ayrıca, M ön-açık fonksiyonların karakterizasyonlarını elde ettik. Üstelik, M ön eş yapılı dönüşüm kavramını verdik ve bu kavramı karakterize ettik.

Anahtar Kelimeler: **ön-açık küme, yarı açık küme, ön komşuluk, ön açık fonksiyon, M ön açık fonksiyon, M ön eş yapılı dönüşüm.**

Preliminaries: Let (X,τ) or, simply X denote a topological space. For any subset $A\subset X$. $Int(A)=A^{\circ}$ and $Cl(A)=A^{-}$ denote the interior and closure of A respectively.

Definition 1.1.[1] Let X be a topological space and S be a subset of X. S is said to be pre-open if $S\subset Int(Cl(S))$. The family of all pre-open sets in X will be denoted by PO(X).

Remark 1.1.Every open set is pre-open set. But the converse not true. As the following example illustrates.

Example 1.1. Let X={a,b,c}. Define $\tau = \{X,\emptyset,\{a\},\{b,c\}\}$ where τ is a topology on X. We show that for $\{b\}\subset X$ subset, $\{b\}\subset\{b\}^{-\circ}$

$$\kappa = \{\emptyset,X,\{b,c\},\{a\}\}$$

is the set of closed sets for the topology τ. Then $\{b\}^{-\circ} = \{b,c\}$. Since $\{b\}\subset \{b,c\}$, {b} is pre-open set, that is, $\{b\}\subset\{b\}^{-\circ}$. But {b} set is not open set.

Definition 1.2.[2] Let X be a topological space and A be a subset of X. A subset A is said to be semi-open if there exists an open set U of X such that $U\subset A\subset U^{-}$ The complement of a semi-open set is called semi-closed set.

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Remark 1.2. A open set is pre-open set if and only if this pre-open set is semi-closed set.
Definition 1.3.[5] Let X be a topological space and F be a subset of X. F is said to be pre-closed if $Cl(Int(F)) \subset F$.
Remark 1.4. Every closed set is pre-closed set. But the converse not true. As the following example illustrates.
Example 1.2. Let X={a,b,c}. Define $\tau = \{X,\emptyset,\{a\},\{b,c\}\}$ where τ is a topology on X. We show that for $\{b\} \subset X$ subset, $\{b\}^{\circ -} \subset \{b\}$

$$\kappa = \{\emptyset, X, \{b,c\}, \{a\}\}$$

is the set of closed sets for the topology τ. Then $\{b\}^{\circ -} = \emptyset$. Since $\emptyset \subset \{b\}$, thus, {b} is pre-closed set, that is, $\{b\}^{\circ -} \subset \{b\}$. But {b} set is not closed set.
Remark 1.5. A closed set is pre- closed set if and only if this pre- closed set is semi-open set.
Definition 1.4.[4] Let x be a point of a topological space X. Subset is called a pre-neigbourhood of x in X if there exists $A \in PO(X)$ such that $x \in A \subset U$
U is called a pre-neigbourhood of x $\Leftrightarrow \exists A \in PO(X) \ni x \in A \subset U$
Definition 1.5.[1] Let X and Y be topological space. The function $f : X \to Y$ is pre-open function if the image of each open set in X is pre-open in Y.
Theorem 1.1. Let X and Y be two topological spaces.The open function is pre-open function if and only if each subset A of X, is semi closed set in Y.
Proof. $\Rightarrow$ Let $T \subset X$ be any open subset. Since f is open function, f(T) set is open set. Since every open set is pre-open set, then the image under mapping f of any open T is pre-open, by Definition 1.5, we have that f function is pre-open function.
$\Leftarrow$ Consider any be open $A \subset X$. We show that f is open function. By the hypothesis, since f(A) is semi-closed set,
$(f(A))^{-\circ} \subset f(A)$ (1)
By the hypothesis, since f is pre-open function, then the image under mapping by f of open set A is pre-open set. That is,

$$(f(A)) \subset (f(A))^{-\circ} \qquad (2)$$

By (1) and (2) statements,

$$f(A) = (f(A))^{-\circ} \qquad (3)$$

By (3) statement, take the interior of both side $(f(A))^{\circ} = ((f(A))^{-\circ})^{\circ}$ and hence

$$(f(A))^{\circ} = (f(A))^{-\circ}$$

By (3) statement, $(f(A))^{\circ} = f(A)$, then f(A) is open set. Consequently, f function is open function.
Theorem 1.2. Let X and Y be any topological spaces. Then the function f^{-1} is a.c.H. if and only if the function $f : X \to Y$ is pre-open function.
Proof. $\Rightarrow$ Let $G \subset X$ be any open subset. Then $(f^{-1})^{-1}(G) \subset (f^{-1})^{-1}(X)$. However $(f^{-1})^{-1}(G) = f(G) \subset Y$. By the hypothesis, since the function f^{-1} is a.c.H., $(f^{-1})^{-1}(G)$ is pre-open set, that is, f(G) is pre-open set. Hence, by Definition 1.5., f is pre-open function.
$\Leftarrow$ Let $G \subset X$ be any open set. Since f is pre-open function, f(G) is pre-open set in Y. Since f(G) set is written by form

$$f(G) = (f^{-1})^{-1}(G)$$

hence f^{-1} is a.c.H. (see Definition of a.c.H.[1])
Theorem 1.3.[1] Let X and Y be any topological spaces. For a function $f : X \to Y$ the following properties are equivalent:
(I) The function f is pre-open.
(II) For each point x in X and each neighbourhood $U \subset X$ with $x \in U$, there is a pre-open set $f(x) \in V \subset Y$ such that $V \subset f(U)$.
Proof. (I) $\Rightarrow$ (II) Let $x \in X$ and U be a open neighbourhood of x. According to the hypothesis, since f is pre-open function, $f(x) \in f(U)$ is pre-open set and since every pre-open is pre-neighbourhood (see

[4],) $f(U)\subset Y$ is pre-neighbourhood. Then by Definition 1.4, there exists a pre-open set V in Y such that $f(x)\in V\subset Y$.

(II) $\Rightarrow$ (I) Let $x\in X$ and $x\in U$ be any subset in X. By the hypothesis, there is a pre-open set V in Y such that

$$f(x)\in V\subset f(U) \qquad (1)$$

From here if we take, the closure of both side at first, the interior of both side later, we get

$$V^{-\circ}\subset (f(U))^{-\circ} \qquad (2)$$

Since V is pre-open set, by Definition 1.1,

$$V\subset V^{-\circ} \qquad (3)$$

By (2) and (3) statements,

$$V\subset V^{-\circ}\subset (f(U))^{-\circ}$$

then

$$f(x)\in V\subset (f^{-1}(U))^{-\circ} \qquad (4)$$

From (1) statement $x\in f(U)$ and by (4), $x\in (f(U))^{-\circ}$ Hence $f(U)\subset (f(U))^{-\circ}$ and by Definition 1.1, f(U) is pre-open set. Since the image under mapping of open set is pre-open set, f is pre-open function.

Theorem 1.4. Let X and Y be any topological spaces. For a function $f : X \rightarrow Y$ the following properties are equivalent:

(I) The function f is pre-open.

(II) For any point x in X the image under mapping of f of every neighbourhood U of x is a pre-neighbourhood of f(x).

(III) For each point x in X and each neighbourhood $U\subset X$ of x, there is a pre- neighbourhood $V\subset Y$ of x such that $V\subset f(U)$.

Proof. (I) $\Rightarrow$ (II) For any point $x\in X$, U is a neighbourhood of x. By Definition of neighbourhood, there is a open set $T\subset X$ such that

$$x\in T\subset U \qquad (1)$$

from here, take the image under mapping by f of both side,

$$f(x)\in f(T)\subset f(U)$$

by the hypothesis, since f is pre-open function, f (T) is pre-open set. Then, by Definition 1.4, f(U) is a pre-neighbourhood of point f(x).

(II) $\Rightarrow$ (III) Let $x\in X$ and U be a neighbourhood of x. By (II), f(U) is a pre-neighbourhood of f(x). According to Definition 1.4, there exists a pre-neighbourhood V such that $f(x)\in V\subset f(U)$. Since every pre-open set is pre-neighbourhood, V set is a pre-neighbourhood of point f(x).

(III) $\Rightarrow$ (I) Let x be a point in X. Suppose that U is a pre-neighbourhood of point x. By (III), there is a pre-neighbourhood V such that $f(x)\in V\subset f(U)$. Then f(U) is a pre-neighbourhood of point f(x) as well. Hence by Definition 1.4, there exists a pre-neighbourhood W such that $f(x)\in W\subset f(U)$. Therefore, by Definition 1.5, we have that f is a pre-open function.

Theorem 1.5 Let $f : (X,\tau) \rightarrow (Y,\upsilon)$ be surjective, pre-open function with G(f) closed. Then Y space is T_2 - space.

Proof. Let y and w be distinct points in Y. Since f is surjective function, then there are distinct points x and z in X such that f(x) = y and f(z) = w. Since $(x,w)\notin G_f$ and $G_f \subset XxY$ is closed, there exists open sets U and V containing x and w respectively, such that

$$f(U)\cap V = \varnothing \qquad (1)$$

hence $f(U)\subset Y-V$. From here take the closure of both side, $(f(U))^{-}\subset (Y-V)^{-}$. Since Y-V is closed, Y-V $=(Y-V)^{-}$. Hence

$$(f(U))^{-}\subset Y-V \qquad (2)$$

Since f is pre-open function, f(U) is pre-open set in Y. That is,

$$f(x)\in f(U)\subset (f(U))^{-\circ} \qquad (3)$$

By (2) statement, we have

$$(f(U))^{-\circ} \subset (Y-V)^{\circ} \qquad (4)$$

By (3) statement, $f(x) \in (f(U))^{-\circ}$, According to (3) and (4) statements, $f(x) \in f(U) \subset (Y-V)^{\circ}$, that is, there exists open set $(Y-V)^{\circ}$ containing y. $V \cap (Y-V)^{\circ} = \varnothing$. (Y,υ) space is T_2 - (Hausdorff) space.

Theorem 1.6. Let X and Y be any topological spaces. Then the function $f : X \to Y$ is pre-open function if and only if each subset $B \subset X$, $f(B^{\circ}) \subset (f(B))^{op}$

Proof. $\Rightarrow$ Let $B \subset X$ be any subset. B° is open set in (X,τ) and by hypothesis, f is pre-open function, $f(B^{\circ})$ is pre-open set. It is always true that $B^{\circ} \subset B$. From here $f(B^{\circ}) \subset f(B)$ and then if we take the pre-interior of both side, we have $(f(B^{\circ}))^{op} \subset (f(B))^{op}$ [see [6]]. Since $f(B^{\circ})$ is pre-open set and the pre-interior of pre-open set is itself, thus

$$f(B^{\circ}) \subset (f(B))^{op}$$

$\Leftarrow$ Let $B \subset X$ be any subset. Since B is open set, $B^{\circ} = B$. By the hypothesis $f(B^{\circ}) \subset (f(B))^{op}$ hence we get

$$f(B) \subset (f(B))^{op} \qquad (1)$$

In addition,

$$(f(B))^{op} \subset f(B) \text{ (see. [3])} \qquad (2)$$

By (1) and (2) statements, we have $f(B) = (f(B))^{op}$. Then f (B) set is pre-open set. According to the Definition 1.5, we get that f is pre-open function.

Theorem 1.7. Let X and Y be any topological spaces. For a function $f : X \to Y$ the following properties are equivalent:

(I) The function f is pre-open.

(II) For each subset A of X, $f(A^{\circ}) \subset (f(A))^{op}$

(III) For each $B \in \beta$ set, f(B) set is pre-open.

Proof. (I) $\Rightarrow$ (II) This is seen from the Theorem 1.6.

(II) $\Rightarrow$ (III) For each $B \in \beta$ set, since $\beta \subset \tau$, $B \in \tau$. From here B is open set, written $B^{\circ} = B$. By the hypothesis, $f(B^{\circ}) \subset (f(B))^{op}$ and

$$f(B) \subset (f(B))^{op} \qquad (1)$$

In addition,

$$f(B))^{op} \subset f(B) \text{ (see. [3])} \qquad (2)$$

By (1) and (2) statements, $f(B) = (f(B))^{op}$. Thus, f (B) set is pre-open set.

(III) $\Rightarrow$ (I) Consider any open subset $A \subset X$. Since β is a basis, A is the union of members of β, that is,

$$A = \bigcup_{i \in I} B_i \qquad (3)$$

According to (III) statement, $f(B_i)$ is pre-open set. In (3) statement, take the image under mapping by f of both side, we have

$$f(A) = f\left(\bigcup_{i \in I} B_i\right) = \bigcup_{i \in I} f(B_i)$$

Since the union of pre-open sets is again a pre-open set (see. [3]), f(A) is a pre-open set. By the Definition 1.5, f is pre-open function.

Definition 1.6.[3]. Let X and Y be topological space. The function $f : X \to Y$ is M pre-open function if the image of each pre-open set in X is pre-open in Y.

Some properties of M pre-open mappings are given in the following theorem:

Theorem 1.8. Let X and Y be any topological spaces. For a function $f : X \to Y$ the following properties are equivalent:

(I) The function f is M pre-open.

(II) For each point x in X and each pre-neighbourhood $U \subset X$ with $x \in U$, there is a pre-open set $f(x) \in V \subset Y$ such that $V \subset f(U)$.

Proof. (ı) ⇒ (ıı) Let $x \in X$ and U be a pre-open neighbourhood of x. According to the hypothesis, since f is M pre-open function, $f(x) \in f(U)$ is pre-open set and since every pre-open is pre-neighbourhood (see. [4]) $f(U) \subset Y$ subset is pre-open neighbourhood. Then, there exists a pre-open set V in Y such that $f(x) \in V \subset Y$.

(ıı) ⇒ (ı) Let $x \in X$ and $x \in U$ be any pre-open subset in X. By the hypothesis, there is a pre-open set V in Y such that

$$f(x) \in V \subset f(U) \qquad (1)$$

From here if we take, the closure of both side at first, the interior of both side later, we get

$$V^{-\circ} \subset (f(U))^{-\circ} \qquad (2)$$

Since V is pre-open set, by Definition 1.1,

$$V \subset V^{-\circ} \qquad (3)$$

By (2) and (3) statements,

$$V \subset V^{-\circ} \subset (f(U))^{-\circ}$$

then

$$f(x) \in V \subset (f^{-1}(U))^{-\circ} \qquad (4)$$

From (1) statement $x \in f(U)$ and by (4), $x \in (f(U))^{-\circ}$ Hence $f(U) \subset (f(U))^{-\circ}$ and by Definition 1.1, f(U) is pre-open set. Thus, since the image under mapping of pre-open set is pre-open set, f is M pre-open function.

Theorem 1.9. Let X and Y be any topological spaces. For a function $f : X \to Y$ the following properties are equivalent:

(ı) The function f is M pre-open.

(ıı) For any point x in X the image under mapping of f of every pre-neighbourhood U of x is a pre-neighbourhood of f(x).

(ııı) For each point x in X and each pre-neighbourhood $U \subset X$ of x, there is a pre-neighbourhood $V \subset Y$ of x such that $V \subset f(U)$.

Theorem 1.10. Let X and Y be any topological spaces. Then the function $f : X \to Y$ is M pre-open function if and only if each subset $B \subset X$,

$f(B^{\circ p}) \subset (f(B))^{\circ p}$

Proof. ⇒ Let $B \subset X$ be any subset. İt is always true that $B^{\circ p} \subset B$. From here if we take the image under mapping by f of both side, $f(B^{\circ p}) \subset f(B)$. $B^{\circ p}$ is pre-open set and by the hypothesis since f is M pre-open function, $f(B^{\circ})$ is pre-open set. In $f(B^{\circ p}) \subset f(B)$ statement, if we take the pre-interior of both side, we have $(f(B^{\circ p}))^{\circ p} \subset (f(B))^{\circ p}$. $f(B^{\circ})$ is pre-open set and the pre-interior of pre-open set is itself, thus $f(B^{\circ p})^{\circ p} = f(B^{\circ p})$ then, $f(B^{\circ p}) \subset (f(B))^{\circ p}$

⇐ Let $B \subset X$ be any pre-open subset. Since B is pre-open set, $B^{\circ p} = B$ (see. [6]). By the hypothesis $f(B^{\circ p}) \subset (f(B))^{\circ p}$, hence we get

$$f(B) \subset (f(B))^{\circ p} \qquad (1)$$

In addition,

$$(f(B))^{\circ p} \subset f(B) \text{ (see. [3])} \qquad (2)$$

By (1) and (2) statements, we have $f(B) = (f(B))^{\circ p}$. Then f (B) set is pre-open set. According to the Definition 1.6, we get that f is M pre-open function.

Theorem 1.11. Let X and Y be any topological spaces. Then the function f^{-1} is M pre-continuous if and only if the function $f : X \to Y$ is M pre-open function.

Proof. ⇒ Let $A \subset X$ be any pre-open subset. From here if we take the inverse image under mapping of f of both side, $(f^{-1})^{-1}(A) \subset (f^{-1})^{-1}(X)$. However $(f^{-1})^{-1}(A) = f(A) \subset Y$. By the hypothesis, since the function f^{-1} is M pre-continuous, $(f^{-1})^{-1}(A)$ is pre-open set, that is, f(A) is pre-open set. Hence, by Definition 1.5, f is M pre-open function.

⇐ Let $A \subset X$ be any pre-open set. Since f is M pre-open function, f(A) is pre-open set in Y. Since f(A) set is written by form

$$f(A) = (f^{-1})^{-1}(A)$$

hence, f^{-1} is M pre-continuous function.

Definition 1.7.[3]. Let X and Y be topological space. The function $f : X \to Y$ is M pre-closed function if the image of each pre-closed set in X is pre-closed in Y.

Theorem 1.12. Let X and Y be any topological spaces. Then the function $f : X \to Y$ is M pre-closed function if and only if each subset $A \subset X$, $(f(A))^{-p} \subset f(A^{-p})$.

Proof. ⇒ Let $A \subset X$ be any subset. İt is always true that $A \subset A^{-p}$[5]. From here if we take the image under mapping by f of both side, $f(A) \subset f(A^{-p})$. B^{op} is pre-closed set and by the hypothesis since f is M pre-closed function, $f(A^{-p})$ is pre-open set. In $f(A) \subset f(A^{-p})$ statement, if we take the pre-closure of both side, we have $(f(A))^{-p} \subset (f(A^{-p}))^{-p}$. $f(A^{-p})$ is pre-closed set and the pre-closure of pre-closed set is itself, thus $(f(A^{-p}))^{-p} = f(A^{-p})$ then, $(f(A))^{-p} \subset f(A^{-p})$.

⇐ Let $F \subset X$ be any pre-closed subset. We show that f(F) is pre-closed set, that is, $f(F)=(f(F))^{-p}$. Since F is pre-closed set,

$$F = F^{-p} \text{ (see [5]).} \qquad (1)$$

From here if we take the image under mapping by f of both side,

$$f(F) = f(F^{-p}) \qquad (2)$$

by (III) statement,

$$(f(F))^{-p} \subset f(F^{-p})$$

by (1) and (2) statements, we have

$$(f(F))^{-p} \subset f(F) \qquad (3)$$

Moreover

$$f(F) \subset (f(F))^{-p} \qquad (4)$$

According to (3) and (4) statements, f (F) set is pre-closed set. Consequently, f is M pre-closed function.

Theorem 1.13. Let X and Y be any topological spaces. Then the function f^{-1} is M pre-continuous if and only if the function $f : X \to Y$ is M pre-closed function.

Proof. ⇒ Let $F \subset X$ be any pre-closed subset. From here if we take the inverse image under mapping of f of both side, $(f^{-1})^{-1}(F) \subset (f^{-1})^{-1}(X)$. However $(f^{-1})^{-1}(F) = f(F) \subset Y$. By the hypothesis, since the function f^{-1} is M pre-continuous, $(f^{-1})^{-1}(F)$ is pre-closed set, that is, f(F) is pre-closed set. Hence, by Definition 1.7, f is M pre-closed function.

⇐ Let $F \subset X$ be any pre-closed set. Since f is M pre- closed function, f(F) is pre-closed set in Y. Since f(F) set is written by form

$$f(F) = (f^{-1})^{-1}(F)$$

hence, f^{-1} is M pre-continuous function.

Definition 1.8. Let X and Y be topological space. A mapping $f : X \to Y$ is called M pre-homeomorphism if there exists a bijective mapping f such that f and f^{-1} are M pre-continuous functions.

Remark 1.5. For M pre-homeomorphism concept, by Theorem 1.11 and Theorem 1.13, we give the following thorem;

Theorem 1.14. Let $f : X \to Y$ be bijective function. Then the function f is M pre-homeomorphism if and only if f is M pre-continuous and M pre-open function.

Theorem 1.15. Let $f : X \to Y$ be bijective function. Then the function f is M pre-homeomorphism if and only if f is M pre-continuous and M pre-closed function.

Now, we give a criter related to M pre-homeomorphism function in the following;

Theorem 1.16. Let X and Y be any topological spaces. Then the function $f : X \to Y$ is M pre-homeomorphism function if and only if each subset $A \subset X$,

$(f(A))^{-p} = f(A^{-p})$

Proof. ⇒ Let f be M pre-homeomorphism. By Definition 1.8, f is M pre-continuous. Then for each $A \subset X$ subset

$$f(A^{-p}) \subset (f(A))^{-p} [5] \qquad (1).$$

According to Definition 1.8., since f^{-1} is M pre-continuous function by Theorem 1.13, f is M pre-closed function. According to Theorem 1.12,

$$(f(A))^{-p} \subset f(A^{-p}) \qquad (2).$$

Therefore by (1) and (2) statements, for each $A \subset X$ subset,

$$f(A^{-p}) = (f(A))^{-p}$$

⇐ For each $A \subset X$ subset, if $f(A^{-p}) = (f(A))^{-p}$, by Theorem 1.12, f is M pre-closed function and f is M pre-continuous [4]. Consequently, by Theorem 1.15, f is M pre-homeomorphism.

Theorem 1.17. Let $f : (X,\tau_1) \to (Y,\tau_2)$ and $g : (Y,\tau_2) \to (Z,\tau_3)$ be any to mappings. If f is a pre-open and g is a M pre-open, then $gof : (X,\tau_1) \to (Z,\tau_3)$ is a pre-open function.

Proof. Let A be open subset of topological space X. By the hipotezix, since f is a pre-open function, $f(A) \subset Y$ is pre-open set. Since g is M pre-open function, by Definition 1.6, $g(f(A)) \subset Z$ is pre-open set, that is, $g(f(A))=(gof)(A)$ is pre-open set. Therefore gof is pre-open function.

Theorem 1.18. Let $f : (X,\tau_1) \to (Y,\tau_2)$ and $g : (Y,\tau_2) \to (Z,\tau_3)$ be any to mappings. If f is a pre-continuous and surjective function and $gof : (X,\tau_1) \to (Z,\tau_3)$ is a M pre-open function, then g is a pre-open function.

Proof. Let A be open subset of topological space X. By the hipotezix, since f is pre-continuous function, $f^{-1}(A) \subset X$ is pre-open set. Since gof is M pre-open function, the image under mapping of gof of $f^{-1}(A)$ set is pre-open set, that is, $(gof)(f^{-1}(A)) \subset Z$ is pre-open set.

$$(gof)(f^{-1}(A))=g(f(f^{-1}(A)))$$

and since f is surjective function, $(gof)(f^{-1}(A))=g(A)$. Consequently, g is pre-open function.

Theorem 1.19. Let $f : (X,\tau_1) \to (Y,\tau_2)$ and $g : (Y,\tau_2) \to (Z,\tau_3)$ be any to mappings. If gof is a pre-open and g is injective and M pre-continuous, then $f : (X,\tau_1) \to (Y,\tau_2)$ is a pre-open function.

Proof. Let G be open subset of topological space X. By the hipotezix, since gof is a pre-open function, $(gof)(G)=g(f(G))$ is pre-open set. Since g is injective and M pre-continuous function,

$$g^{-1}(g(f(G))) = g^{-1}g(f(G)) = f(G) \subset Y$$

f(G) is pre-open set. Consequently, f is pre-open function.

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